

# Position dependent energy level shifts of an accelerated atom in the presence of a boundary

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## Abstract

We consider a uniformly accelerated atom interacting with a vacuum electromagnetic field in the presence of an infinite conducting plane boundary and calculate separately the contributions of vacuum fluctuations and radiation reaction to the atomic energy level shift. We analyze in detail the behavior of the total energy shift in three different regimes of the distance in both the low acceleration and high acceleration limits. Our results show that, in general, an accelerated atom does not behave as if immersed in a thermal bath at the Unruh temperature in terms of the atomic energy level shifts, and the effect of the acceleration on the atomic energy level shifts may in principle become appreciable in certain circumstances, although it may not be realistic for actual experimental measurements. We also examine the effects of the acceleration on the level shifts when the acceleration is of the order of the transition frequency of the atom and we find some features differ from what was obtained in the existing literature.

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## I. INTRODUCTION

An important prediction of quantum field theory is the existence of quantum fluctuations of electromagnetic fields even in vacuum. These quantum fluctuations lead to a number of observable effects such as the Lamb shift, and the Casimir and Casimir-Polder forces (see, Ref. [1] for an extensive review). All these effects have been observed experimentally. The Casimir-Polder force was originally studied between a neutral electric polarizable atom at rest and a conducting plane in vacuum by Casimir and Polder [2], and later a variety of situations have been considered, for example, the case of an atom near the surface of a dielectric slab [3] or a nanostructure [4], and the case of the atom-surface system in or out of thermal equilibrium [5–7]. In addition, the case of atoms on accelerated trajectories has also been investigated [8–12]. Our interest in the Casimir-Polder force associated with accelerated atoms is two-fold. First, such studies may shed some light on our understanding of the Unruh effect [13], on which controversy still exists [14], and, second, the CP force of an accelerated atom, which arises as a result of the change of atomic energy level shifts, may provide a new possibility to detect the Unruh effect.

In this regard, let us note that the Casimir-Polder interaction energy between an accelerated atom in interaction with the vacuum electromagnetic fields and an infinite conducting plane boundary, which is equal to the energy shift of the atom caused by the presence of the boundary, has recently been studied, and it is found that the effect of acceleration is not purely thermal [12]. In that work, a generalization of the formalism suggested by Dalibard, Dupont-Roc and Cohen-Tannoudji [20, 21](DDC), which allows a separate calculation of contributions of vacuum fluctuation and radiation reaction to the energy level shift is employed, and the result is however expressed as an integral of a very complicated function which makes it hard both for analytical examination of the behavior of the level shifts in different distance regimes and for numerical analysis. In the present paper, we plan to revisit the problem. We have been able to obtain a result of the energy level shift of the accelerated atom, which is almost in a closed form except for a part which is a integral of a much simpler function. This enables us to make a thorough comparison, in different distance regimes, with

the result of the case of a static atom in a thermal bath found by us using the same formalism [5] to see whether the accelerated atom behaves as if immersed in a thermal bath in terms of atomic energy level shifts <sup>1</sup>. More importantly, the numerical analysis of our result at an acceleration of the transition frequency of the atom yields conclusions that differ from those obtained in Ref. [12]. For example, we find, contrary to the conclusion in Ref. [12], that the energy level shift of an atom with an acceleration of the typical transition frequency in the vicinity of an infinite conducting plane is smaller than that of a static one when the distance is at the order of  $10^{-6}\text{m}$ , among others.

The paper is organized as follows, in Sec. II, using the DDC formalism, we separately calculate both the contributions of vacuum fluctuations and radiation reaction to the position dependent energy shifts of an atom on a uniformly accelerated trajectory, which gives rise to the Casimir-Polder force on the accelerated atom. In Sec. III, we analyze in detail the behavior of the energy shift of a ground-state atom in three different regimes of the distance in both the low acceleration and high acceleration limits. In addition, with the method of numerical analysis, we also discuss the behavior of the energy shift for an atom with a typical acceleration necessary to observe the Unruh effect. Comparing our results with those of a static atom immersed in a thermal bath, we can see explicitly the difference between the energy shift of an accelerated atom and that of a thermal one. Finally, we will conclude in Sec. IV with a summary of results obtained.

## II. VACUUM FLUCTUATIONS AND RADIATION REACTION CONTRIBUTIONS TO THE ENERGY SHIFTS OF AN ACCELERATED ATOM IN THE PRESENCE OF A BOUNDARY

We consider a uniformly accelerated two-level atom in interaction with the vacuum electromagnetic fields in a flat spacetime with an infinite conducting plane boundary. Let us note that for a fully realistic treatment, one may need to consider a multilevel atom. Using the

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<sup>1</sup> Let us note that it has been demonstrated that the accelerated atom near a boundary behaves differently from the inertial one immersed in a thermal bath in terms of the spontaneous excitation rate [15–19].

DDC formalism and following the procedures that have been shown in Refs. [5, 10, 12, 21], we will calculate separately the contributions of vacuum fluctuations and radiation reaction to the energy shifts of a two-level atom. To be self-contained, we will first review the general formalism. Employing the Hamiltonian of the atom-field system that has been given in Ref [5], one can write down the Heisenberg equations of motion for the dynamical variables of the atom and field. The solutions of the equations of motion can be split into the two parts: a free part, which is present even in the absence of the coupling, and which we will denote with the superscript  $f$ , and a source part, which is caused by the interaction of the atom and field, and which we will denote with the superscript  $s$ . We assume that the initial state of the field is the Minkowski vacuum  $|0\rangle$ , while the atom is in the state  $|b\rangle$ . To identify the contributions of vacuum fluctuations and radiation reaction to the radiative energy shifts of our accelerated atoms, we choose a symmetric ordering between the atom and the electric field variables and consider the effects of  $E^f$  (corresponding to the effect of vacuum fluctuations) and  $E^s$  (corresponding to the effect of radiation reaction) separately in the Heisenberg equations of an arbitrary atomic observable. Then taking the average in the vacuum state of the electromagnetic field, one can obtain the effective Hamiltonians

$$H_{vf}^{eff}(\tau) = -\frac{i}{2} \int_{\tau_0}^{\tau} d\tau' C_{ij}^F(x(\tau), x(\tau')) [\mu_i^f(\tau), \mu_j^f(\tau')] , \quad (1)$$

$$H_{rr}^{eff}(\tau) = -\frac{i}{2} \int_{\tau_0}^{\tau} d\tau' \chi_{ij}^F(x(\tau), x(\tau')) \{\mu_i^f(\tau), \mu_j^f(\tau')\} , \quad (2)$$

where  $\mu_i$  ( $\mu_j$ ) is a component of the atomic electric dipole moment,  $\tau$  is the proper time, and  $x(\tau)$  is the stationary trajectory of the atom. Here we take a perturbation treatment up to order  $\mu^2$ , and use  $[ , ]$  and  $\{ , \}$  to denote the commutator and anticommutator. The subscript “ $vf$ ” and “ $rr$ ” stand respectively for the contributions of vacuum fluctuations and radiation reaction. The statistical functions  $C_{ij}^F$  and  $\chi_{ij}^F$  are defined as

$$C_{ij}^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | \{E_i^f(x(\tau)), E_j^f(x(\tau'))\} | 0 \rangle , \quad (3)$$

$$\chi_{ij}^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | [E_i^f(x(\tau)), E_j^f(x(\tau'))] | 0 \rangle , \quad (4)$$

which are also called the symmetric correlation function and the linear susceptibility of the field. Taking the expectation value of Eqs. (1) and (2) in the atom's initial state  $|b\rangle$ , we can obtain the vacuum fluctuations and radiation reaction contributions to the radiative energy shifts of the atom's level  $|b\rangle$ ,

$$(\delta E_b)_{vf} = -i \int_{\tau_0}^{\tau} d\tau' C_{ij}^F(x(\tau), x(\tau')) (\chi_{ij}^A)_b(\tau, \tau') , \quad (5)$$

$$(\delta E_b)_{rr} = -i \int_{\tau_0}^{\tau} d\tau' \chi_{ij}^F(x(\tau), x(\tau')) (C_{ij}^A)_b(\tau, \tau') , \quad (6)$$

where  $(C_{ij}^A)_b$  and  $(\chi_{ij}^A)_b$ , the symmetric correlation function and the linear susceptibility of the atom, are defined as

$$(C_{ij}^A)_b(\tau, \tau') = \frac{1}{2} \langle b | \{ \mu_i^f(\tau), \mu_j^f(\tau') \} | b \rangle , \quad (7)$$

$$(\chi_{ij}^A)_b(\tau, \tau') = \frac{1}{2} \langle b | [ \mu_i^f(\tau), \mu_j^f(\tau') ] | b \rangle , \quad (8)$$

which are just characterized by the atom itself. For the explicit forms, the statistical functions of the atom can be written as

$$(C_{ij}^A)_b(\tau, \tau') = \frac{1}{2} \sum_d [ \langle b | \mu_i(0) | d \rangle \langle d | \mu_j(0) | b \rangle e^{i\omega_{bd}(\tau-\tau')} + \langle b | \mu_j(0) | d \rangle \langle d | \mu_i(0) | b \rangle e^{-i\omega_{bd}(\tau-\tau')} ] , \quad (9)$$

$$(\chi_{ij}^A)_b(\tau, \tau') = \frac{1}{2} \sum_d [ \langle b | \mu_i(0) | d \rangle \langle d | \mu_j(0) | b \rangle e^{i\omega_{bd}(\tau-\tau')} - \langle b | \mu_j(0) | d \rangle \langle d | \mu_i(0) | b \rangle e^{-i\omega_{bd}(\tau-\tau')} ] , \quad (10)$$

where  $\omega_{bd} = \omega_b - \omega_d$  and the sum extends over a complete set of atomic states.

In order to calculate the statistical functions of the electric field, Eqs. (5) and (6), we will firstly consider the two point function of the four potential,  $A^\mu(x)$ , which can be obtained by the method of images. At a distance  $z$  from the boundary, we find in the laboratory frame,

$$D^{\mu\nu}(x, x') = \langle 0 | A^\mu(x) A^\nu(x') | 0 \rangle = D_{free}^{\mu\nu}(x - x') + D_{bnd}^{\mu\nu}(x, x') , \quad (11)$$

where

$$D_{free}^{\mu\nu}(x-x') = \frac{\eta^{\mu\nu}}{4\pi^2[(t-t'-i\varepsilon)^2 - (x-x')^2 - (y-y')^2 - (z-z')^2]} , \quad (12)$$

is the two point function in the free space and

$$D_{bnd}^{\mu\nu}(x, x') = -\frac{\eta^{\mu\nu} + 2n^\mu n^\nu}{4\pi^2[(t-t'-i\varepsilon)^2 - (x-x')^2 - (y-y')^2 - (z+z')^2]} , \quad (13)$$

represents the correction induced by the presence of the conducting boundary. Here  $\varepsilon \rightarrow +0$ ,  $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ , the unit normal vector  $n^\mu = (0, 0, 0, 1)$ , and the subscript “bnd” stands for the part induced by the presence of the boundary. From Eq. (11), we can get the electric field two point function in the laboratory frame,

$$\langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle = \langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle_{free} + \langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle_{bnd} , \quad (14)$$

where

$$\begin{aligned} \langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle_{free} &= \frac{1}{4\pi^2}(\delta_{ij}\partial_0\partial'_0 - \partial_i\partial'_j) \\ &\times \frac{1}{(x-x')^2 + (y-y')^2 + (z-z')^2 - (t-t'-i\varepsilon)^2} , \end{aligned} \quad (15)$$

and

$$\begin{aligned} \langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle_{bnd} &= -\frac{1}{4\pi^2}[(\delta_{ij} - 2n_i n_j)\partial_0\partial'_0 - \partial_i\partial'_j] \\ &\times \frac{1}{(x-x')^2 + (y-y')^2 + (z+z')^2 - (t-t'-i\varepsilon)^2} . \end{aligned} \quad (16)$$

Here  $\partial'$  denotes the differentiation with respect to  $x'$ . Then the statistical functions of the electric field can be found from Eq. (14) as a sum of the free space part and the boundary-dependent part.

In the present paper, we are interested in the energy shifts of atomic levels caused by the presence of the plane boundary. These energy level shifts are position dependent and give rise to the Casimir-Polder force acting on the atom. So, subtracting the free space part in Eqs. (5) and (6), we can obtain the contributions of vacuum fluctuations and radiation reaction to the boundary-dependent energy shifts of atomic levels,

$$(\delta E_b)_{vf}^{bnd} = -i \int_{\tau_0}^{\tau} d\tau' (C_{ij}^F)_{bnd}(x(\tau), x(\tau')) (\chi_{ij}^A)_b(\tau, \tau') , \quad (17)$$

$$(\delta E_b)_{rr}^{bnd} = -i \int_{\tau_0}^{\tau} d\tau' (\chi_{ij}^F)_{bnd}(x(\tau), x(\tau')) (C_{ij}^A)_b(\tau, \tau') . \quad (18)$$

Therefore the total energy shifts caused by the presence of a conducting plane boundary is

$$(\delta E_b)_{tot}^{bnd} = (\delta E_b)_{vf}^{bnd} + (\delta E_b)_{rr}^{bnd} . \quad (19)$$

We assume that the conducting plane boundary is located at  $z = 0$  in space, and the two-level atom is being uniformly accelerated in the  $x$ -direction with a proper acceleration  $a$  at a distance  $z$  from the boundary. The atom's trajectory can be described with the proper time  $\tau$  by

$$t(\tau) = \frac{1}{a} \sinh a\tau , \quad x(\tau) = \frac{1}{a} \cosh a\tau , \quad y(\tau) = y_0 , \quad z(\tau) = z . \quad (20)$$

With this trajectory, we can calculate the boundary-dependent two point function of the electric field in the proper reference frame of the atom with a Lorentz transformation in Eq. (16),

$$\begin{aligned} \langle 0 | E_i(x(\tau)) E_j(x(\tau')) | 0 \rangle_{bnd} = & -\frac{a^4}{16\pi^2} \frac{1}{[\sinh^2 \frac{a}{2}(u - i\varepsilon) - a^2 z^2]^3} \\ & \times \left\{ \left[ \delta_{ij} - 2n_i n_j + 2az(n_i k_j + k_i n_j) \right] \sinh^2 \frac{au}{2} \right. \\ & \left. + a^2 z^2 \left[ \delta_{ij} \cosh^2 \frac{au}{2} + (\delta_{ij} - 2k_i k_j) \sinh^2 \frac{au}{2} \right] \right\} , \quad (21) \end{aligned}$$

where  $u = \tau - \tau'$ , the unit vector  $k^\mu = (0, 1, 0, 0)$ , and it points along the direction of acceleration. Therefore, from Eqs. (3) and (4), the boundary-dependent statistical functions of the electric field can be expressed as

$$\begin{aligned} (C_{ij}^F)_{bnd}(x(\tau), x(\tau')) = & -\frac{a^4}{32\pi^2} \left( \frac{1}{[\sinh^2 \frac{a}{2}(u - i\varepsilon) - a^2 z^2]^3} + \frac{1}{[\sinh^2 \frac{a}{2}(u + i\varepsilon) - a^2 z^2]^3} \right) \\ & \times \left\{ \left[ \delta_{ij} - 2n_i n_j + 2az(n_i k_j + k_i n_j) \right] \sinh^2 \frac{au}{2} \right. \\ & \left. + a^2 z^2 \left[ \delta_{ij} \cosh^2 \frac{au}{2} + (\delta_{ij} - 2k_i k_j) \sinh^2 \frac{au}{2} \right] \right\} , \quad (22) \end{aligned}$$

and

$$\begin{aligned}
(\chi_{ij}^F)_{bnd}(x(\tau), x(\tau')) = & -\frac{a^4}{32\pi^2} \left( \frac{1}{[\sinh^2 \frac{a}{2}(u - i\varepsilon) - a^2 z^2]^3} - \frac{1}{[\sinh^2 \frac{a}{2}(u + i\varepsilon) - a^2 z^2]^3} \right) \\
& \times \left\{ [\delta_{ij} - 2n_i n_j + 2az(n_i k_j + k_i n_j)] \sinh^2 \frac{au}{2} \right. \\
& \left. + a^2 z^2 [\delta_{ij} \cosh^2 \frac{au}{2} + (\delta_{ij} - 2k_i k_j) \sinh^2 \frac{au}{2}] \right\}. \quad (23)
\end{aligned}$$

Here one can see that for these statistical functions of the electric field, the diagonal components (the  $xx$ ,  $yy$  and  $zz$  components), and the off-diagonal  $xz$  component are nonzero. With the statistical functions given above, we will use the residue theorem to calculate separately the contributions of vacuum fluctuations and radiation reaction to boundary-dependent energy shifts. To be generic, we assume that the accelerated atom is arbitrarily polarized. Thus we need to consider all the nonzero statistical functions in our calculations.

Now we take the  $xx$  component for an example to show how the calculations are to be carried out. Substituting the statistical functions (9), (10), (22) and (23) into the general formulas (17) and (18) and letting  $i = j = x$ , we can obtain

$$\begin{aligned}
(\delta E_b)_{vf,xx}^{bnd} = & \frac{ia^4}{64\pi^2} \sum_d |\langle b | \mu_x(0) | d \rangle|^2 \int_0^\infty du (e^{i\omega_{bd}u} - e^{-i\omega_{bd}u}) \\
& \times \left( \frac{\sinh^2 \frac{au}{2} + a^2 z^2}{[\sinh^2 \frac{a(u-i\varepsilon)}{2} - a^2 z^2]^3} + \frac{\sinh^2 \frac{au}{2} + a^2 z^2}{[\sinh^2 \frac{a(u+i\varepsilon)}{2} - a^2 z^2]^3} \right), \quad (24)
\end{aligned}$$

and

$$\begin{aligned}
(\delta E_b)_{rr,xx}^{bnd} = & \frac{ia^4}{64\pi^2} \sum_d |\langle b | \mu_x(0) | d \rangle|^2 \int_0^\infty du (e^{i\omega_{bd}u} + e^{-i\omega_{bd}u}) \\
& \times \left( \frac{\sinh^2 \frac{au}{2} + a^2 z^2}{[\sinh^2 \frac{a(u-i\varepsilon)}{2} - a^2 z^2]^3} - \frac{\sinh^2 \frac{au}{2} + a^2 z^2}{[\sinh^2 \frac{a(u+i\varepsilon)}{2} - a^2 z^2]^3} \right), \quad (25)
\end{aligned}$$

where we have extended the range of integration to infinity for sufficiently long times  $\tau - \tau_0$ . For convenience, we take the atom to be in its ground state. In order to evaluate the integral in Eqs. (24) and (25), we use the residue theorem and consider the contour integral along path  $C_1$  and  $C_2$  in Fig. 1. With a definition of the atomic static scalar polarizability

$$\alpha_0 = \sum_j \alpha_j = \sum_j \frac{2|\langle b | \mu_j(0) | d \rangle|^2}{3\omega_0}, \quad (26)$$



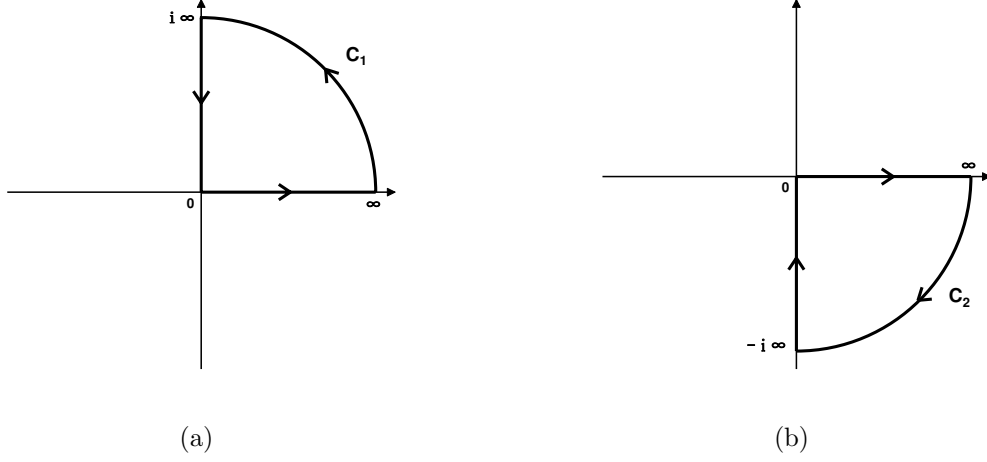


FIG. 1: Integration path of Eqs. (24) and (25).

the contribution of vacuum fluctuations to the position-dependent energy shift of the ground state is calculated to be,

$$(\delta E_-)_{vf,xx}^{bnd} = -\frac{3\omega_0\alpha_x}{128\pi} \left[ \left( 1 + \frac{2}{e^{2\pi\omega_0/a} - 1} \right) f_{xx}(\omega_0, z, a) - g_{xx}(\omega_0, z, a) \right], \quad (27)$$

and that of radiation reaction

$$(\delta E_-)_{rr,xx}^{bnd} = \frac{3\omega_0\alpha_x}{128\pi} f_{xx}(\omega_0, z, a), \quad (28)$$

where we have defined

$$f_{xx}(\omega_0, z, a) = \frac{4z^2\omega_0^2(1+a^2z^2) - 4a^4z^4 - 2a^2z^2 - 1}{z^3(1+a^2z^2)^{5/2}} \cos\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right) - \frac{2\omega_0(1+4a^2z^2)}{z^2(1+a^2z^2)^2} \sin\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right), \quad (29)$$

and

$$g_{xx}(\omega_0, z, a) = \frac{4a^4}{\pi} \int_0^\infty du \frac{\sin^2 \frac{au}{2} - a^2z^2}{(\sin^2 \frac{au}{2} + a^2z^2)^3} e^{-\omega_0 u}. \quad (30)$$

Other nonzero components can be computed similarly. In summary, for the case of an accelerated ground-state atom in the vicinity of an infinite conducting plane, the contribution of vacuum fluctuations to the position-dependent energy shift is given by

$$(\delta E_-)_{vf}^{bnd} = -\frac{3\omega_0\sqrt{\alpha_i\alpha_j}}{128\pi} \left[ \left( 1 + \frac{2}{e^{2\pi\omega_0/a} - 1} \right) f_{ij}(\omega_0, z, a) - g_{ij}(\omega_0, z, a) \right], \quad (31)$$

while the contribution of radiation reaction is

$$(\delta E_-)_{rr}^{bnd} = \frac{3\omega_0\sqrt{\alpha_i\alpha_j}}{128\pi} f_{ij}(\omega_0, z, a) . \quad (32)$$

Here summation over repeated indices,  $i, j$ , is implied. The nonzero functions  $f_{xx}(\omega_0, z, a)$  and  $g_{xx}(\omega_0, z, a)$  are given by Eqs. (29) and (30), and the others are

$$f_{yy}(\omega_0, z, a) = \frac{4z^2\omega_0^2(1+a^2z^2)-1}{z^3(1+a^2z^2)^{3/2}} \cos\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right) - \frac{2\omega_0(1+2a^2z^2)}{z^2(1+a^2z^2)} \sin\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right) , \quad (33)$$

$$f_{zz}(\omega_0, z, a) = -\frac{2+a^2z^2[5-4z^2\omega_0^2(1+a^2z^2)]}{z^3(1+a^2z^2)^{5/2}} \cos\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right) - \frac{2\omega_0(2+a^2z^2+2a^4z^4)}{z^2(1+a^2z^2)^2} \sin\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right) , \quad (34)$$

$$f_{xz}(\omega_0, z, a) = \frac{a[1+4a^2z^2+4z^2\omega_0^2(1+a^2z^2)]}{z^2(1+a^2z^2)^{5/2}} \cos\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right) + \frac{2a\omega_0(1-2a^2z^2)}{z(1+a^2z^2)^2} \sin\left(\frac{2\omega_0 \sinh^{-1}(az)}{a}\right) , \quad (35)$$

$$g_{yy}(\omega_0, z, a) = \frac{4a^4}{\pi} \int_0^\infty du \frac{\sin^2 \frac{au}{2} - a^2z^2 \cos(au)}{(\sin^2 \frac{au}{2} + a^2z^2)^3} e^{-\omega_0 u} , \quad (36)$$

$$g_{zz}(\omega_0, z, a) = -\frac{4a^4}{\pi} \int_0^\infty du \frac{\sin^2 \frac{au}{2} + a^2z^2 \cos(au)}{(\sin^2 \frac{au}{2} + a^2z^2)^3} e^{-\omega_0 u} , \quad (37)$$

$$g_{xz}(\omega_0, z, a) = \frac{8a^5z}{\pi} \int_0^\infty du \frac{\sin^2 \frac{au}{2}}{(\sin^2 \frac{au}{2} + a^2z^2)^3} e^{-\omega_0 u} . \quad (38)$$

One can see clearly that both vacuum fluctuations and radiation reaction depend on the atom's acceleration. This differs from the thermal corrections to the energy shifts of a static atom [5], where radiation reaction is independent of the temperature. A remarkable feature worth noting is that the position-dependent energy shift for an atom polarized in the  $x-z$  plane gets an extra contribution associated with the functions  $f_{xz}$  and  $g_{xz}$  as compared with

that of the static case [5, 22]. This is in qualitative agreement with the result in Ref. [12], and it opens up the possibility of observing the effect of acceleration of the atomic energy level shifts using atoms with anisotropic polarization as pointed out in Ref. [12]. Adding up the contributions of vacuum fluctuations and the radiation reaction, we can obtain the total position-dependent energy shift of the ground state

$$(\delta E_-)_{tot}^{bnd} = -\frac{3\omega_0\sqrt{\alpha_i\alpha_j}}{128\pi} \left[ \frac{2}{e^{2\pi\omega_0/a} - 1} f_{ij}(\omega_0, z, a) - g_{ij}(\omega_0, z, a) \right]. \quad (39)$$

If the atom's acceleration equals to zero, we can recover the results in Ref [22], which give the energy level shifts of a static atom near a conducting plane in vacuum. Now one can see that our expression for the position-dependent energy shift is in a simpler and almost closed form as compared to that given in Ref. [12] where the energy shift is written as integral of a very complicated function (refer to Eqs.(29) and (30) of Ref. [12]). So, our result seems to be easier to handle in terms of both analytical and numerical analysis. In what follows, we examine the behaviors of the energy shift in various circumstances and compare our results with those in the case of a static atom in a thermal bath at the the Unruh temperature related to the acceleration [5].

### III. DISCUSSION

Now we examine the behaviors of the position-dependent energy shift in the limits of low acceleration (when the atom's acceleration is much smaller than the transition frequency of the atom,  $a \ll \omega_0$ ) and high acceleration (when the atom's acceleration is much larger than the transition frequency of the atom,  $a \gg \omega_0$ ), analogously with the low- and high-temperature limits  $T \ll \omega_0$  and  $T \gg \omega_0$ , in Ref. [5]. With the method of numerical analysis, we also analyze the case when  $a \sim \omega_0$ .

#### A. Low-acceleration limit

In the low-acceleration limit  $a \ll \omega_0$ , we can identify the distance between the atom and the boundary into three different regimes: the short distance, where the distance  $z$  is so small

that  $az \ll \omega_0 z \ll 1$ ; the intermediate distance, where  $az \ll 1 \ll \omega_0 z$ ; and the long distance, where the distance  $z$  is so large that  $\omega_0 z \gg az \gg 1$ . For these three distinct regimes, we will discuss the behavior of the position-dependent energy shift of a ground state atom.

Let us first consider the short distance regime, where  $az \ll \omega_0 z \ll 1$ . In this case, the contributions of vacuum fluctuations and radiation reaction to the energy shift of a ground state can be expressed as

$$(\delta E_-)_{vf}^{bnd} \approx -\frac{1}{4\pi} \left[ -\frac{3\omega_0^2 \alpha_z}{4\pi z^2} + \frac{3a\omega_0^2 \sqrt{\alpha_x \alpha_z}}{4\pi z} - \frac{\omega_0^2 (a^2 + \omega_0^2)}{\pi} \ln(2\omega_0 z) (\alpha_x + \alpha_y - \alpha_z) \right], \quad (40)$$

and

$$(\delta E_-)_{rr}^{bnd} \approx -\frac{1}{4\pi} \left[ \frac{3\omega_0 (\alpha_x + \alpha_y + 2\alpha_z)}{32z^3} - \frac{3a^2 \omega_0}{64z} \left( \alpha_x + 3\alpha_y + 32\omega_0^2 z^2 \alpha_z + \frac{2}{az} \sqrt{\alpha_x \alpha_z} \right) \right]. \quad (41)$$

Here the contribution of radiation reaction is much larger than that of vacuum fluctuations and plays the dominant role in the total energy shift. Note that the acceleration induced correction term associated with the atomic polarization in the  $z$  direction in Eq. (41) is much smaller than that in Eq. (40). So the total energy shift containing the acceleration corrections can be written approximately as

$$(\delta E_-)_{tot}^{bnd} \approx \frac{1}{4\pi} \left[ -\frac{3\omega_0 (\alpha_x + \alpha_y + 2\alpha_z)}{32z^3} + \frac{3a^2 \omega_0}{64z} \left( \alpha_x + 3\alpha_y + \frac{64\omega_0 z \ln(2\omega_0 z)}{3} \alpha_z + \frac{2}{az} \sqrt{\alpha_x \alpha_z} \right) \right]. \quad (42)$$

The first (leading) term on the righthand side of Eq. (42) is just the energy shift of a static atom interacting with a vacuum electromagnetic field near an infinite conducting plane [22]. The total energy shift is always negative, and both the leading term and the acceleration correction terms depend on the direction of the atom's polarization. For an isotropically polarized atom, the off-diagonal  $xz$  component contributes the biggest acceleration correction term and the position-dependent energy shift can be written as

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{1}{4\pi} \left( \frac{\omega_0 \alpha_0}{8z^3} - \frac{\omega_0 \alpha_0 a}{32z^2} \right), \quad (43)$$

where the first (leading) term is just the result of an inertial atom in the vacuum, and the acceleration correction term is proportional to the atom's acceleration. These two terms are opposite in sign. With the Unruh temperature  $T_U = a/(2\pi)$ , Eq. (43) becomes

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{1}{4\pi} \left( \frac{\omega_0 \alpha_0}{8z^3} - \frac{\pi \omega_0 \alpha_0}{16z^2} T_U \right), \quad (44)$$

which grows linearly with the Unruh temperature. Let us recall the position-dependent energy shift of a static atom at low temperature and in the short distance regime [5], where the thermal correction is dominated by the term proportional to  $T^4$ . Therefore, in this case the acceleration effect is not equal to the thermal effect, although the total energy shifts agree in the leading order.

Then in the intermediate distance regime, where  $az \ll 1 \ll \omega_0 z$ , we find, approximately, the contributions of vacuum fluctuations and radiation reaction,

$$\begin{aligned} (\delta E_-)_{vf}^{bnd} \approx & -\frac{1}{4\pi} \left\{ \left[ \frac{3\omega_0^3}{8z} \left( \alpha_x + \alpha_y - \frac{1}{2z^2\omega_0^2} \alpha_z \right) \right. \right. \\ & - \frac{3\omega_0^3 a^2 z}{16} \left( 3\alpha_x + \alpha_y - 2\alpha_z - \frac{2\sqrt{\alpha_x \alpha_z}}{az} \right) \Big] \cos(2\omega_0 z) \\ & - \left[ \frac{3\omega_0^2(\alpha_x + \alpha_y + 2\alpha_z)}{16z^2} + \frac{3\omega_0^2 a^2}{16} \left( 2\alpha_x + \alpha_y - 3\alpha_z - \frac{\sqrt{\alpha_x \alpha_z}}{az} \right) \right] \sin(2\omega_0 z) \\ & \left. + \frac{3}{8\pi z^4} (\alpha_x + \alpha_y + \alpha_z) + \frac{3a^2}{8\pi \omega_0^2 z^4} \left( \frac{2\alpha_x}{\omega_0^2 z^2} - \alpha_y - \alpha_z - \frac{\sqrt{\alpha_x \alpha_z}}{az} \right) \right\}, \quad (45) \end{aligned}$$

and

$$\begin{aligned} (\delta E_-)_{rr}^{bnd} \approx & \frac{1}{4\pi} \left\{ \left[ \frac{3\omega_0^3}{8z} \left( \alpha_x + \alpha_y - \frac{1}{2z^2\omega_0^2} \alpha_z \right) \right. \right. \\ & - \frac{3\omega_0^3 a^2 z}{16} \left( 3\alpha_x + \alpha_y - 2\alpha_z - \frac{2\sqrt{\alpha_x \alpha_z}}{az} \right) \Big] \cos(2\omega_0 z) \\ & \left. - \left[ \frac{3\omega_0^2(\alpha_x + \alpha_y + 2\alpha_z)}{16z^2} + \frac{3\omega_0^2 a^2}{16} \left( 2\alpha_x + \alpha_y - 3\alpha_z - \frac{\sqrt{\alpha_x \alpha_z}}{az} \right) \right] \sin(2\omega_0 z) \right\}, \quad (46) \end{aligned}$$

where both have terms which are oscillating functions of  $z$ . However, when we add up these two contributions, the oscillating components cancel out, and we find

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{1}{4\pi} \left[ \frac{3}{8\pi z^4} (\alpha_x + \alpha_y + \alpha_z) + \frac{3a^2}{8\pi \omega_0^2 z^4} \left( \frac{2\alpha_x}{\omega_0^2 z^2} - \alpha_y - \alpha_z - \frac{\sqrt{\alpha_x \alpha_z}}{az} \right) \right]. \quad (47)$$

Here the first term plays the dominant role in the position-dependent energy shift, and it is in agreement with that of a static atom placed far from the boundary in the vacuum [22]. For an isotropically polarized atom, the acceleration correction comes mainly from the off-diagonal  $xz$  component under the condition  $az \ll 1 \ll \omega_0 z$ , and the position-dependent energy shift becomes

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{1}{4\pi} \left( \frac{3\alpha_0}{8\pi z^4} - \frac{\alpha_0 a}{8\pi \omega_0^2 z^5} \right) = -\frac{1}{4\pi} \left( \frac{3\alpha_0}{8\pi z^4} - \frac{\alpha_0}{4\omega_0^2 z^5} T_U \right). \quad (48)$$

Here the position-dependent energy shift is still negative, and grows linearly with the Unruh temperature, while in the case of a static atom immersed in a thermal bath, the temperature correction is proportional to  $T^4$  in the intermediate distance regime for the low temperature limit [5].

Finally, it is time to examine the case of the atom in the long distance regime ( $\omega_0 z \gg az \gg 1$ ). In order to compare the accelerated case with the thermal case in this regime, let us first recall the energy shift of an isotropically polarized atom in a thermal bath in the long distance regime in the low temperature limit [5],

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{1}{4\pi} \frac{\alpha_0 T}{4z^3}. \quad (49)$$

If an accelerated atom were to behave the same as a static one immersed in a thermal bath at the Unruh temperature  $T_U = a/(2\pi)$ , we would expect that the energy shift of the accelerated atom should be proportional to the atom's acceleration. But, in fact, the energy shift (39) can also be approximated, in the long-distance regime for the low-acceleration limit, by Eq. (48), where the acceleration just induces a very small correction, and this correction term, although linear in the Unruh temperature, is not the leading term as in the thermal case. This shows that the behavior of an accelerated atom in the long distance regime is completely different from that of the static one immersed in a thermal bath at the Unruh temperature.

## B. High-acceleration limit

Now let us turn our attention to the high-acceleration limit, where we assume that the atom's acceleration is much larger than the transition frequency of the atom,  $a \gg \omega_0$ . In this limit the contribution of vacuum fluctuations to the energy shift of the ground state, Eq. (31), can be written approximately as

$$(\delta E_-)_{vf}^{bnd} = -\frac{3\omega_0\sqrt{\alpha_i\alpha_j}}{128\pi} \left( \frac{a}{\pi\omega_0} f_{ij}(\omega_0, z, a) - g_{ij}(\omega_0, z, a) \right). \quad (50)$$

As in the low-acceleration limit, we will analyze the behavior of the energy-level shift in three different regimes. Let us first deal with the case when the atom is so close to the boundary that  $\omega_0 z \ll az \ll 1$  (the short distance regime). It then follows that

$$(\delta E_-)_{vf}^{bnd} \approx \frac{1}{4\pi} \frac{3a}{32\pi z^3} (\alpha_x + \alpha_y + 2\alpha_z - az\sqrt{\alpha_x\alpha_z}), \quad (51)$$

and

$$(\delta E_-)_{rr}^{bnd} \approx -\frac{1}{4\pi} \left[ \frac{3\omega_0(\alpha_x + \alpha_y + 2\alpha_z)}{32z^3} - \frac{3\omega_0 a \sqrt{\alpha_x\alpha_z}}{32z^2} - \frac{3\omega_0 a^2(\alpha_x + 3\alpha_y)}{64z} - \frac{45\omega_0 a^4 z \alpha_z}{128} \right]. \quad (52)$$

Note that the contribution of vacuum fluctuations comes mainly from the acceleration correction terms, while the acceleration just gives very small corrections to that of the radiation reaction. Under the high acceleration condition ( $a \gg \omega_0$ ), the contribution of vacuum fluctuations is much larger than that of radiation reaction and plays the dominant role in the boundary-dependent energy shift. For an isotropically polarized atom, the total energy shift becomes

$$(\delta E_-)_{tot}^{bnd} \approx \frac{1}{4\pi} \left( \frac{\alpha_0 a}{8\pi z^3} - \frac{\alpha_0 a^2}{32\pi z^2} \right) = \frac{1}{4\pi} \left( \frac{\alpha_0}{4z^3} T_U - \frac{\pi\alpha_0}{8z^2} T_U^2 \right), \quad (53)$$

where the first (leading) term is linear in the acceleration of the atom and is the same as the corresponding result of an inertial atom immersed in a thermal bath at the Unruh temperature [5]. But the subleading term that comes from the  $xz$  component is absent in the thermal case. So, in terms of the energy level shifts, the accelerated behaves as if immersed in a thermal bath at the Unruh temperature in the leading order. This differs from the

corresponding result in the scalar field case in the high acceleration limit, where the energy shift is independent of the acceleration and differs completely from a static atom immersed in a thermal bath at the Unruh temperature [23].

If we increase, in the high acceleration limit ( $a \gg \omega_0$ ), the distance  $z$  between the atom and the conducting boundary, such that  $az \gg 1$ , the contributions of vacuum fluctuations and radiation reaction can be approximated as

$$\begin{aligned}
(\delta E_-)_{vf}^{bnd} \approx & -\frac{1}{4\pi} \left\{ -\frac{3\alpha_x}{8\pi z^4} \left[ \left(1 - \frac{\omega_0^2}{a^2}\right) \cos \frac{2\omega_0}{a} \ln(2az) + \frac{2\omega_0}{a} \sin \frac{2\omega_0}{a} \ln(2az) - 1 \right] \right. \\
& + \frac{3\omega_0^2 \alpha_y}{8\pi z^2} \left[ \left(1 - \frac{1}{4\omega_0^2 a^2 z^4}\right) \cos \frac{2\omega_0}{a} \ln(2az) - \frac{a}{\omega_0} \sin \frac{2\omega_0}{a} \ln(2az) + \frac{1}{a^2 z^2} + \frac{1}{4\omega_0^2 a^2 z^4} \right] \\
& + \frac{3\omega_0^2 \alpha_z}{8\pi z^2} \left[ \left(1 - \frac{5}{4\omega_0^2 a^2 z^4}\right) \cos \frac{2\omega_0}{a} \ln(2az) - \frac{a}{\omega_0} \sin \frac{2\omega_0}{a} \ln(2az) + \frac{1}{a^2 z^2} + \frac{5}{4\omega_0^2 a^2 z^4} \right] \\
& \left. + \frac{3\omega_0^2 \sqrt{\alpha_x \alpha_z}}{8\pi a z^3} \left[ \cos \frac{2\omega_0}{a} \ln(2az) - \frac{a}{\omega_0} \sin \frac{2\omega_0}{a} \ln(2az) - \frac{1}{\omega_0^2 z^2} \right] \right\}, \tag{54}
\end{aligned}$$

and

$$\begin{aligned}
(\delta E_-)_{rr}^{bnd} \approx & \frac{1}{4\pi} \left\{ -\frac{3\omega_0 \alpha_x}{8a z^4} \left[ \left(1 - \frac{\omega_0^2}{a^2}\right) \cos \frac{2\omega_0}{a} \ln(2az) + \frac{2\omega_0}{a} \sin \frac{2\omega_0}{a} \ln(2az) \right] \right. \\
& + \frac{3\omega_0^3 \alpha_y}{8a z^2} \left[ \left(1 - \frac{1}{4\omega_0^2 a^2 z^4}\right) \cos \frac{2\omega_0}{a} \ln(2az) - \frac{a}{\omega_0} \sin \frac{2\omega_0}{a} \ln(2az) \right] \\
& + \frac{3\omega_0^3 \alpha_z}{8a z^2} \left[ \left(1 - \frac{5}{4\omega_0^2 a^2 z^4}\right) \cos \frac{2\omega_0}{a} \ln(2az) - \frac{a}{\omega_0} \sin \frac{2\omega_0}{a} \ln(2az) \right] \\
& \left. + \frac{3\omega_0^3 \sqrt{\alpha_x \alpha_z}}{8a^2 z^3} \left[ \cos \frac{2\omega_0}{a} \ln(2az) - \frac{a}{\omega_0} \sin \frac{2\omega_0}{a} \ln(2az) \right] \right\}, \tag{55}
\end{aligned}$$

where the approximation of  $\sinh^{-1}(az) \sim \ln(2az)$  for  $az \gg 1$  is used. Here both the contributions of vacuum fluctuations and radiation reaction contain terms that are oscillating functions of the atom's acceleration and the distance  $z$ , and the amplitudes of the oscillating functions in Eq. (55) are much smaller than those in Eq. (54). Therefore, the energy-level shift of the ground-state atom exhibits an oscillatory behavior, which is typical of the energy-level shift of an excited state. This can be understood as a result of the fact that the ground-state atoms have a nonvanishing possibility to absorb thermal photons to transit to the excited states in the high-temperature limit, and it actually brings in an interesting issue of the thermal average of energy-level shifts of an atom in thermal equilibrium. Let us



note that this issue, which we leave for future research, can be addressed using our results in the present paper and the spontaneous excitation rate found in Ref. [18] in a similar way as what has been done in the case of a static atom in a thermal bath [5].

If we further assume that  $\frac{2\omega_0}{a} \ln(2az) \ll 1$ , which is easily satisfied in the limits  $a \gg \omega_0$  and  $az \gg 1$ , since the logarithm is a very slowly varying function, the above results can be simplified to

$$(\delta E_-)_{vf}^{bnd} \approx -\frac{1}{4\pi} \left\{ \frac{3\omega_0^2}{8\pi a^2 z^4} \left[ 1 - 4 \ln(2az) + 2(\ln(2az))^2 \right] \alpha_x + \frac{3\omega_0^2}{8\pi z^2} [1 - 2 \ln(2az)] \left( \alpha_y + \alpha_z + \frac{1}{az} \sqrt{\alpha_x \alpha_z} \right) \right\}, \quad (56)$$

and

$$(\delta E_-)_{rr}^{bnd} \approx \frac{1}{4\pi} \left\{ -\frac{3\omega_0}{8az^4} \alpha_x + \frac{3\omega_0^3}{8az^2} [1 - 2 \ln(2az)] \left( \alpha_y + \alpha_z + \frac{1}{az} \sqrt{\alpha_x \alpha_z} \right) - \frac{3\omega_0}{32a^3 z^6} (\alpha_y + 5\alpha_z - 4az \sqrt{\alpha_x \alpha_z}) \right\}. \quad (57)$$

If the atom is polarized only in the  $x$ -direction, the contribution of radiation reaction is much larger than that of vacuum fluctuations. However, if the atom's polarization is along the  $y$  or  $z$  direction, the ratio of vacuum fluctuations part to radiation reaction part is determined by the magnitude of the quantity  $\omega_0 z$ . If  $\omega_0 z \ll 1$  (the intermediate distance regime), this ratio is indeterminate with an indeterminate quantity  $\omega_0 a^3 z^4$ . Let us note that this disparity between the  $x$  and  $y$  components does not exist in the thermal case, and this is not surprising since the atom is accelerating in the  $x$  direction anyway. If the atom is so far from the boundary that  $\omega_0 z \gg 1$  (the long distance regime), the contributions of vacuum fluctuations associated with the atomic polarization in the  $y$  and  $z$  directions are much larger than that of radiation reaction, and the total energy shift can be written as

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{1}{4\pi} \left\{ \frac{3\omega_0}{8az^4} \alpha_x + \frac{3\omega_0^2}{8\pi z^2} [1 - 2 \ln(2az)] \left( \alpha_y + \alpha_z + \frac{1}{az} \sqrt{\alpha_x \alpha_z} \right) \right\}, \quad (58)$$

which is negative if the atom is polarized in the direction along the atom's acceleration ( $x$  direction) and positive if the atom is polarized in the  $y$  or  $z$  direction. Written in terms of the Unruh temperature, the total energy shift becomes

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{1}{4\pi} \left\{ \frac{3\omega_0}{16\pi z^4 T_U} \alpha_x + \frac{3\omega_0^2}{8\pi z^2} [1 - 2 \ln(4\pi z T_U)] \left( \alpha_y + \alpha_z + \frac{1}{2\pi z T_U} \sqrt{\alpha_x \alpha_z} \right) \right\}, \quad (59)$$

which differs from the corresponding result of a static atom immersed in a thermal bath at the Unruh temperature [5].

### C. The case of $a \sim \omega_0$

Having discussed both the cases of low- ( $a \ll \omega_0$ ) and high-acceleration ( $a \gg \omega_0$ ) limits, we now consider the case when  $a \sim \omega_0$ , since this is probably the typical acceleration necessary to observe the Unruh effect. Although we will mostly resort to numerical method for analysis, using our result which is simpler than that of Ref. [12], we are able to obtain approximate analytical expressions in some special cases.

Let us first examine what happens when the atom is very close to the boundary ( $z\omega_0 \ll 1$ ). Now one can show that

$$(\delta E_-)_{tot}^{bnd} \approx -\frac{3\omega_0}{128\pi} \left[ \frac{1}{z^3}(\alpha_x + \alpha_y + 2\alpha_z) - \frac{a}{z^2} \sqrt{\alpha_x \alpha_z} \right], \quad (60)$$

where the leading term is coincident with the energy shift of a static atom interacting with a vacuum electromagnetic field near an infinite conducting plane [22]. Here, the acceleration correction term comes from the contribution of radiation reaction which is modified by the presence of the acceleration in sharp contrast to the case of a static atom in a thermal bath and it is also a result of the off-diagonal  $xz$  component of  $f$  functions which is absent in the thermal case. It is this off-diagonal term which is unique to the case of an accelerated atom that makes the energy shift smaller than that of a static one.

If the atom is far from the boundary ( $z\omega_0 \gg 1$ ), then one finds

$$\begin{aligned} (\delta E_-)_{tot}^{bnd} \approx & -\frac{3\omega_0}{128\pi} \left\{ \frac{2}{e^{2\pi} - 1} \left[ \left( \frac{2\alpha_x}{\omega_0^3 z^6} + \frac{4\omega_0(\alpha_y + \alpha_z)}{z^2} + \frac{4\sqrt{\alpha_x \alpha_z}}{z^3} \right) \cos(2 \sinh^{-1}(az)) \right. \right. \\ & \left. \left. - \left( \frac{8\alpha_x}{\omega_0 z^4} + \frac{4\omega_0(\alpha_y + \alpha_z)}{z^2} + \frac{4\sqrt{\alpha_x \alpha_z}}{z^3} \right) \sin(2 \sinh^{-1}(az)) \right] \right. \\ & \left. + \frac{2}{\pi \omega_0 z^4} \left( 2\alpha_x + \alpha_y + \alpha_z - \frac{\sqrt{\alpha_x \alpha_z}}{\omega_0 z} \right) \right\}, \quad (61) \end{aligned}$$

which is an oscillating function of  $z$ . The oscillatory behavior here, which is reminiscent of the energy level shift of a static excited state, can again be attributed to the nonvanishing

spontaneous excitation rate of a ground state when  $a \gtrsim \omega_0$ . It is to be noted that if the atom is polarized in the  $y$  and  $z$  directions, the amplitude is proportional to  $1/z^2$ . Recalling the case of a static atom where the energy shift is proportional to  $1/z^4$  when  $z\omega_0 \gg 1$  [22], we see that the energy shift of an accelerated atom with anisotropic polarizability may be much larger than that of the static one when the atom is far from the boundary. This may open up a possibility of an indirect detection of the Unruh effect through the measurements related to the energy-level shifts of accelerating atoms. It is interesting to note that a similar result has been recently obtained for the energy shift between two accelerating atoms in Ref. [24].

Taking  $\omega_0$  to be the typical transition frequency of a hydrogen atom, i.e.,  $\omega_0 \sim 10^{15} \text{ s}^{-1}$ , we have plotted, in Fig. 2, the energy shift as a function of the distance  $z$  on the order of microns for a ground-state atom with isotropic polarizability and for different values of atomic acceleration.

A comparison between the different curves in Fig. 2 shows that the effects of acceleration on the energy shift become appreciable for accelerations on the order of  $10^{23} \text{ m/s}^2$ , and with the increase of the acceleration the absolute values of the energy shift decrease. This is contrary to the conclusion drawn from Fig. 1 in Ref. [12], where the effect of acceleration is found to make the energy shift larger than that of the atom at rest and the absolute values of the energy shift increase with the the increase of the acceleration <sup>2</sup>.

In Fig. 3, we have plotted, on a larger distance scale, the energy shift for a ground-state atom with isotropic polarizability and for different values of atomic acceleration. From the figure, one can see that when  $a \sim 10^{22} \text{ m/s}^2$ , which is one order of magnitude less than  $\omega_0$ , the effect of acceleration makes the energy shift smaller than that of an atom at rest. But the discrepancy between these two energy shifts is very small. This numerical result is in agreement with that in the case of low acceleration limit analytically discussed in the previous section, where we find that the acceleration correction is small and opposite in sign compared with the energy shift of a static atom. After this consistency check, we now move to the more interesting case, i.e., when  $a \sim \omega_0$ . Then the corresponding acceleration  $a \sim 10^{23} \text{ m/s}^2$ .

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<sup>2</sup> Note the SI units are adopted in all the figures of this paper, as opposed to the CGS units used in Ref. [12].

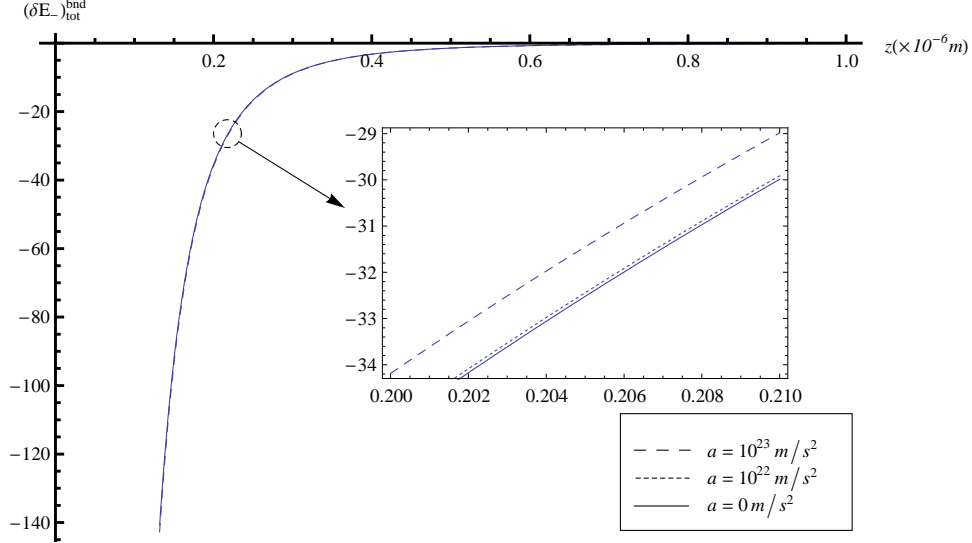


FIG. 2: Energy shifts as a function of the distance  $z$  and for different values of atomic acceleration. Here the typical value of the atomic transition frequency,  $\omega_0 = 10^{15} \text{ s}^{-1}$ , is used and the energy shifts are in the units of  $\alpha_0/(128\pi\epsilon_0)$ .  $\epsilon_0$  is the vacuum dielectric constant. The dashed, dotted and solid lines represent the energy shifts for  $a = 10^{23} \text{ m/s}^2$ ,  $a = 10^{22} \text{ m/s}^2$  and  $a = 0 \text{ m/s}^2$  respectively. For the distance  $z$  on the range of  $0 \sim 1 \times 10^{-6} \text{ m}$ , we can distinguish, by eye, the three lines of different acceleration in the figure from each other. But on an extremely small range, one can see the discrepancy clearly.

Now the Figure shows obvious oscillations of the energy shift when distance  $z \gtrsim 10^{-3} \text{ m}$ . For  $z \gtrsim 10^{-3} \text{ m}$ ,  $z\omega_0 \gg 1$  is satisfied and therefore the energy shift can be approximated by the analytical expressions in Eq. (61). This oscillation gives rise to a clear difference between the energy shift of an accelerated atom and that of a static one. So, on a theoretical front, the effect of the atomic acceleration on the energy shift now becomes appreciable. However, it should be noted that, on an experimental front, a distance of  $\sim 10^{-3} \text{ m}$  appears to be unrealistic for experimental techniques, since all actual measurements of the atom-wall force involve much shorter distances.

In order to show the effect of the acceleration more clearly, let us look at the ratio between the energy shift of an accelerated atom in front of an infinite conducting plane and that of

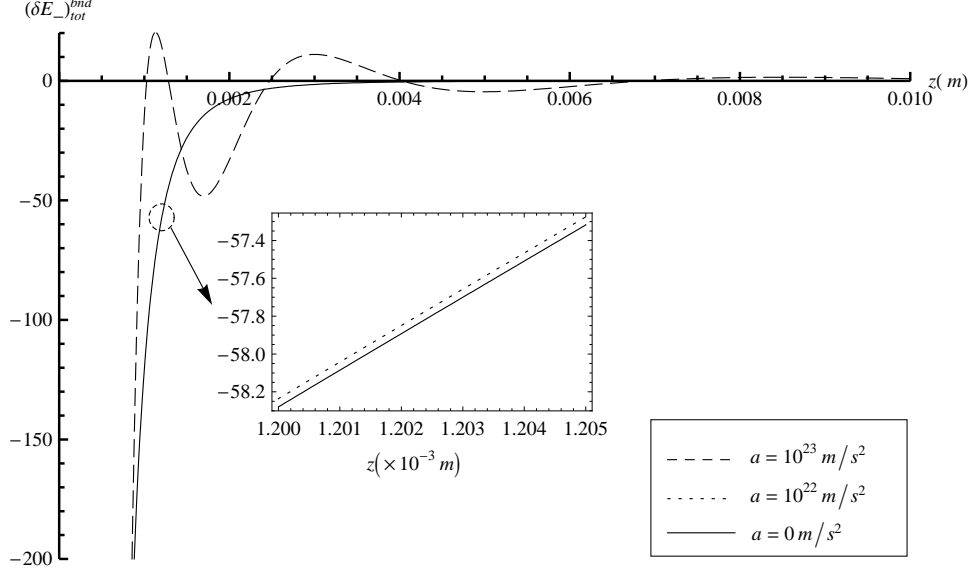


FIG. 3: Energy shifts as a function of the distance  $z$  and for different values of atomic acceleration. Here the typical value of the atomic transition frequency,  $\omega_0 = 10^{15} \text{ s}^{-1}$  is used and the energy shifts are in the units of  $\alpha_0/(128\pi\epsilon_0\omega_0)$ . Note that for the distance  $z$  on the range of  $0 \sim 0.01\text{m}$ , we can not distinguish by eye the lines for  $a = 10^{22} \text{ m/s}^2$  and  $a = 0 \text{ m/s}^2$ . But on an extremely small range, we can see the discrepancy clearly.

a static one  $(\delta E_-)_{tot}^{bnd}(a \neq 0)/(\delta E_-)_{tot}^{bnd}(a = 0)$  for the typical transition frequency of a hydrogen atom,  $\omega_0 \sim 10^{15} \text{ s}^{-1}$ , and the corresponding acceleration  $a \sim 10^{23} \text{ m/s}^2$ . We plot the ratio as a function of the distance  $z$  in Fig. 4, where an isotropic polarizability of the atom is assumed. The sub-figure shows that when the atom is close to the boundary, i.e., when  $z \ll 10^{-6} \text{ m}$ , this ratio approaches to 1, and the energy shift of an accelerated atom approaches to that of a static one. For example, the ratio is  $\sim 0.997$  when  $z \sim 10^{-8} \text{ m}$ . With the increase of the distance  $z$ , we can see clearly the oscillating behaviors of this ratio and the increasing amplitude of the oscillation, although the energy shift of an accelerated atom alone has an oscillating decay. It is interesting to note that the absolute value of this ratio is much larger than 1 for some values of the distance  $z$ , and thus the effect of acceleration on the position-dependent energy shift will be significant, whereas there also exist some values of the distance  $z$  where the ratio equals 1, and, at these distances, the correction of

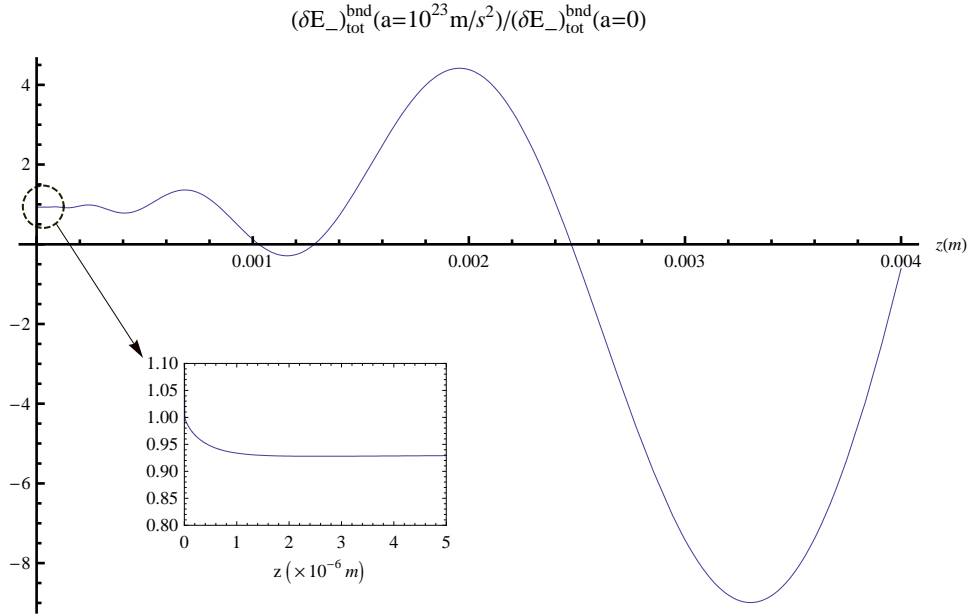


FIG. 4: The ratio between the energy shift of an accelerated atom in front of an infinite conducting plane and that of a static one for acceleration  $a = 10^{23} \text{ m/s}^2$ .

acceleration vanishes. Note that the value of this ratio is smaller than 1 for the distances at the order of  $10^{-6} \text{ m}$ . In other words, the acceleration makes the position-dependent energy shift smaller. This feature differs from that obtained in Ref. [12], where the energy shift of the accelerated atom ( $a = 10^{23} \text{ m/s}^2$ ) is found to be much larger than that of a static one at the same distance scale (refer to Fig. 2 in Ref. [12] and keep in mind that what we actually plot here is the reciprocal of what is plotted there). At the same time, our analysis also reveals an oscillatory behavior of the ratio on a larger distance scale.

Finally, we compare our results with those of a static atom immersed in a thermal bath in the vicinity of an infinite conducting plane [5]. Take the acceleration to be  $a \sim 10^{23} \text{ m/s}^2$ , and the corresponding Unruh temperature is  $T \sim 405 \text{ K}$ . We plot the energy shift of both an accelerated atom and a static one immersed in a thermal bath at the Unruh temperature in Fig. 5, which reveals clearly that the acceleration effect is smaller than the thermal effect. So, the accelerated atom does not behave as if immersed in a thermal bath at the Unruh temperature in terms of the energy level shifts. Like Ref. [12], here we also consider the ratio

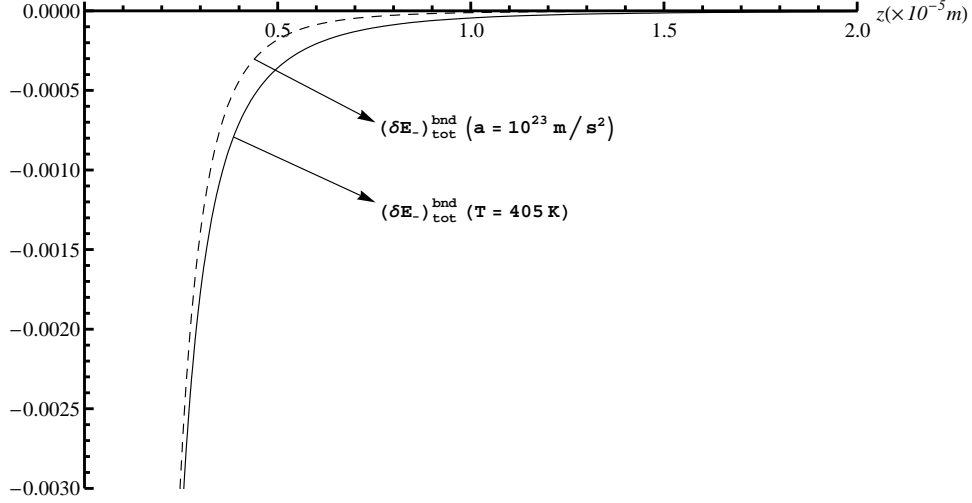


FIG. 5: Energy shifts of both an accelerated atom and a static one immersed in a thermal bath at the corresponding Unruh temperature. Here the typical value of the atomic transition frequency,  $\omega_0 = 10^{15} \text{ s}^{-1}$  is used and the energy shifts are in the units of  $\alpha_0/(128\pi\epsilon_0)$ .

between the energy shift of an accelerated atom in the presence of an infinite conducting plane and that of an atom at rest immersed in a thermal bath at the corresponding Unruh temperature. In Fig. 6, we have plotted the quantity  $(\delta E_-)_{tot}^{bnd}(T = \frac{a}{2\pi})/(\delta E_-)_{tot}^{bnd}(a)$  as a function of the distance  $z$  for an isotropically polarized atom for two different values of acceleration,  $a \sim 10^{23} \text{ m/s}^2$  which satisfies  $a \sim \omega_0$ , and,  $a \sim 10^{22} \text{ m/s}^2$ , which obeys  $a \ll \omega_0$ . These plots indicate that when the atom is very close to the boundary, the ratio  $(\delta E_-)_{tot}^{bnd}(T = \frac{a}{2\pi})/(\delta E_-)_{tot}^{bnd}(a)$  approaches 1 and is independent of the acceleration of the atom. This is expected from our analytical analysis, since in the short distance regime,  $(\delta E_-)_{tot}^{bnd}(T)$  agrees, in the leading order, with  $(\delta E_-)_{tot}^{bnd}(a)$  no matter  $a \sim \omega_0$ , or  $a \ll \omega_0$  (refer to Eq. (60) and Eq. (43)).

However, essentially, the effect of acceleration on the energy shift differs from that of the thermal one. As the distance increases, this ratio grows. The larger the acceleration, the more quickly the ratio grows with the distance. This is also in contrast to the conclusion drawn from Fig. 4 in Ref. [12], where it is found that the ratio may decrease and may become smaller than 1 with the increase of the distance  $z$ .

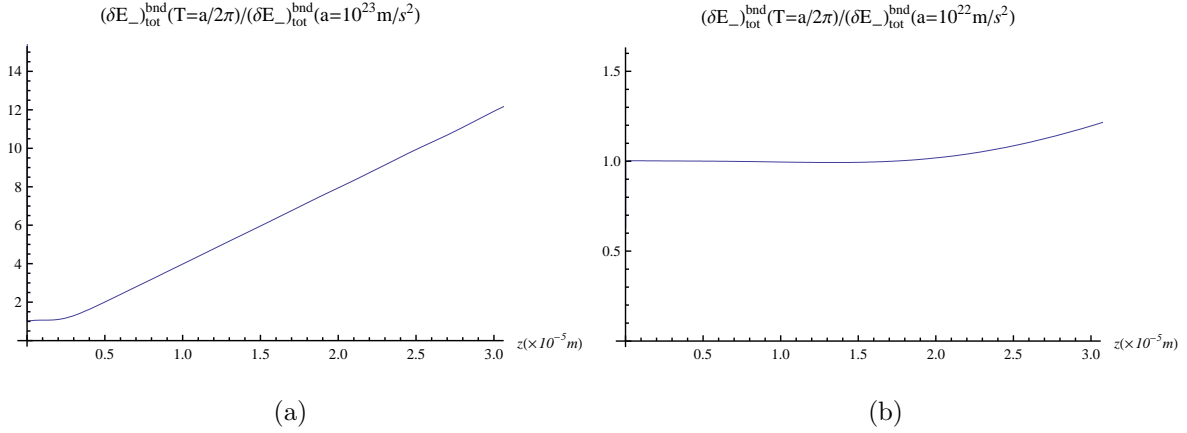


FIG. 6: The ratio between the energy shift of a static atom immersed in a thermal bath at the Unruh temperature  $T = a/(2\pi)$  and that of an accelerated one. Here two different values of acceleration,  $a = 10^{23}\text{m/s}^2$  and  $a = 10^{22}\text{m/s}^2$ , have been considered.

#### IV. CONCLUSIONS

In conclusion, we have calculated separately the contributions of vacuum fluctuations and radiation reaction to the position-dependent energy shift of a uniformly accelerated atom interacting with fluctuating vacuum electromagnetic fields in the vicinity of a plane boundary, which gives rise to the Casimir-Polder force on the accelerated atom. We have analyzed the behaviors of the energy level shifts of the atom in different circumstances.

In the low-acceleration limit where the acceleration is much smaller than the transition frequency of the atom, we found that, in the short and intermediate distance regimes, the energy shift of the accelerated atom is equal to that of a static one immersed in a thermal bath at the Unruh temperature in the leading term. But in the subleading term the acceleration corrections differ from the thermal corrections at the Unruh temperature. In the long-distance regime, even in the leading term, the behavior of an accelerated atom differs completely from that of the static one immersed in a thermal bath at the Unruh temperature.

In the high-acceleration limit, the acceleration correction is equal, in the leading order, to a thermal correction in the short distance regime. However, the off-diagonal  $xz$  component which is absent in the thermal case makes the behavior of an accelerated atom differ from that



of a static atom immersed in a thermal bath at the Unruh temperature. In the intermediate and long distance regimes, the acceleration corrections completely differ from the thermal corrections at the Unruh temperature.

For an acceleration of the order of the transition frequency of the atom, we find, taking the frequency to be that of a hydrogen atom, i.e.,  $\omega_0 \sim 10^{15} \text{ s}^{-1}$ , that the effect of acceleration makes the energy shift smaller than that of an atom at rest when the distance  $z \lesssim 10^{-3} \text{ m}$ , whereas when  $z \gtrsim 10^{-3} \text{ m}$ , the energy shift oscillates significantly as the distance increases. Therefore, there are some distances where the effect of the acceleration on the energy shift is appreciable and some other distances  $z$  where the correction of acceleration vanishes. It should be noted that although the effect of the acceleration on the energy shift may in principle become appreciable, it does not, however, seem to be realistic in actual measurements. Finally, compared with a static atom in a thermal bath, we find that the energy shift of the accelerated atom close to the boundary is smaller than that of the static one at the corresponding Unruh temperature. It is worth pointing out that all these features differ from what is found in Ref. [12].

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